INTENSIFIED DIRECTIVITY OF GAS DISPERSAL AS A RESULT OF RADIATION ENERGY LOSS

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Exact particular solutions of equations of dynamics [1, 2] which hold for special initial conditions (motion with a linear velocity distribution or with uniform deformation), are used extensively to obtain information of good quality about the nature of the dispersal of one-, two-, or three-dimensional gas volumes into a vacuum and quantitative estimates [3-5]. On the basis of solution (2) in [3-5] we considered the adiabatic dispersal of a gas ellipsoid into a vacuum with different ratios of the initial dimensions of this ellipsoid along its axes, including the highly elongated (acicular) or highly compressed (disk-shaped) case. The acceleration is greater along the minor axis than along the major axis since the pressure gradient is greater. The values of the acceleration along the axes become comparable when the dimension of the cloud is of the same order of magnitude along all axes. By that time, however, the velocities along the minor axis can be much higher than along the minor axis and the reserve of thermal energy as a result of adiabatic cooling is insignificant, the pressure is much lower than the initial value, and further dispersal cannot change the velocity ratio. The dispersal, therefore, is directional and the ratios of the ellipsoid directions in the inertial stage of the dispersal are reversed, i.e., the major axis becomes the minor axis.

The directivity of the dispersal intensifies upon transition to a very long needle or a very thin disk (foil) and with increasing adiabatic exponent γ. The explanation for the latter is that the higher γ is, the more rapidly the pressure p and the internal energy e of a unit mass drop as the density ρ decreases. Conversely, as shown in [3] an input of energy reduces the directivity; the ellipsoid heated up in the process becomes rounder.

Interest in problems of gas dispersal into a vacuum has recently been rekindled [1-5] in relation to a number of practical applications [6-8], including the gradual distillation of an evaporating foil by intense laser radiation, high-power electron or ion beams, or a pulse of electric current. At high plasma temperatures the thermal radiation of the plasma becomes a significant factor [9, 10], which can cause more rapid cooling than in the adiabatic case and, therefore, intensify the directivity of the dispersal.

If the pulse from a laser or another external energy source is short (in comparison with the dispersal and radiation time), its role comes down to merely setting the initial temperatures and velocities; without specifying the method of heating the plasma, therefore, we can consider the problem of plasma dispersal into a vacuum with given initial parameters.

In the case of the dispersal of thin foils, heated and/or distilled by the above methods, even in the initial stage of heating and distillation the thickness of the foil is usually less than the mean free path of the emitted radiation. If this is not satisfied from the very beginning, then as the dispersal proceeds the plasma bunch gradually becomes less dense and transparent or semitransparent for its own thermal radiation. The volume energy loss is written as

$$f = 4\varkappa_{\mathrm{P}}\sigma T^4 \tag{1}$$

(f is the energy loss of a unit mass per unit time, T is the temperature, σ is the Stefan-Boltzmann constant, and κ_P is the Planck-averaged mass coefficient of absorption). If the transparency condition is satisfied for some, but not all, wavelengths and part of it is reabsorbed, we can use the approximation of quasi-volume luminescence [9-11], when as before the energy loss is given by (1), but the effective mass coefficient of absorption κ_e appears instead of κ_P: $f = 4\kappa_e\sigma T^4$. The value of κ_e with allowance for reabsorption is determined by solving the spectral transport equation for all frequencies in a uniformly heated layer of gas at the given temperature T and density ρ and for the characteristic dimension R of

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the bunch or its specific mass $m = \rho R$ for the minor axis of the ellipsoid. The quantity κ_e corresponds to the real value of the blackness of the gas. We note that for a plasma that has been ionized repeatedly, but not fully, κ_e can be several orders of magnitude smaller than κ_p [11].

We write the equation for the volume or quasivolume energy loss

$$\partial e/\partial t + p\partial v/\partial t = -f \tag{2}$$

[v is the specific volume ($v = \rho^{-1}$)]. Suppose that f is a power-law function of the thermodynamic parameters:

$$f = Fe^{-\alpha}\rho^{\beta} = Fe^{-\alpha}v^{-\beta}. \tag{3}$$

We write the equation of state as

$$e = pv/(\gamma - 1) \tag{4}$$

(γ is the effective adiabatic exponent).

For aluminum in the range T = 10-120 eV we can assume that $\kappa \sim T^{-3}v^{-2/3}$, with $e \sim T^{3/2}$ [11] and, therefore, in the given case $\alpha \sim -2/3$, $\beta = 2/3$, and $\gamma = 1.2$.

We consider the three-dimensional motion of a gas. The equations of motion and continuity for the Lagrange variables in the Cartesian coordinate system have the form

$$\partial u_i/\partial t + v\partial p/\partial x_i = 0 \; (i = 1,\, 2,\, 3),$$
$$\frac{v(\xi_n,\, t)}{v(\xi_n,\, 0)} = \frac{\partial x_1,\, \partial x_2,\, \partial x_3}{\partial \xi_1,\, \partial \xi_2,\, \partial \xi_3}, \quad \partial x_i/\partial t = u_i, \tag{5}$$

where x_i is the Eulerian coordinate ($x_i = x_i(t, \xi_n)$, n = 1, 2, 3); ξ_i is the Lagrange coordinate of the point ($\xi_i = x_i(0)$); u_i is the velocity of the Lagrangian point ξ_i with coordinates x_i in the Eulerian system. System (2)-(5) can be reduced to a dimensionless form by the transformation

$$u_i = u_* u_i', \quad v = v_* v', \quad e = e_* e',$$
$$t = t_* t', \quad p = p_* p', \quad x_i = x_* x_i', \quad \xi_i = x_* \xi_i', \quad f = f_* f' \tag{6}$$

(the primes label dimensionless quantities). Only three of the eight dimensional factors are independent. We choose the following characteristic parameters as the independent parameters: $x_* = (\xi_i{}^0)_{min}$ is the minimum dimension of the volume under consideration (for a foil, its thickness), p_* is the initial pressure in the plasma, and v_* is the initial specific volume. The other quantities are determined by

$$e_* = p_* v_*, \quad u_* = \sqrt{e_*}, \quad t_* = x_*/u_*, \quad f_* = Fe_*^{-\alpha} v_*^{-\beta}. \tag{7}$$

We thus have the following characteristic quantities: e_* is the internal energy in the initial stage of dispersal, t_* is the gasdynamic time of dispersal along the minor axis without allowance for the radiation loss, u_* is the velocity of the jet after reaching the asymptotic curve, and f_* is the radiation energy loss. We introduce the dimensionless parameter $Q = \frac{f_* t_*}{e_*} = \frac{F t_*}{e_*} \frac{1}{e_*^{\alpha} v_*^{\beta}}$, which is the ratio of the characteristic times t_* of the gasdynamic motion to the luminescence time $t_r = e_*/f_*$.

Let us consider a specific example. Suppose that the dispersal of an aluminum foil with $x_* = 0.3$ mm and $\rho_* = 3\cdot 10^{-3}$ g/cm³ occurs at $T_* = 120$ eV. The blackness $\eta \sim 2\cdot 10^{-3}$ [11]. At $e_* = 10^4$ kJ/g the time $t_r = 6$ nsec. Since $t_* = 2$ nsec, Q = 0.3. The radiation loss in this case will substantially affect the gasdynamic motion. Below we use only the dimensionless variables, omitting the primes. All the equations in system (2)-(5) are invariant under the transformations (6) and (7), except the energy equation (4), which has the form

$$\partial e/\partial t + p\partial v/\partial t = Qe^{-\alpha}v^{-\beta}. \tag{8}$$

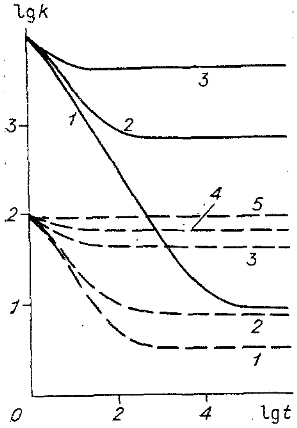

Fig. 1

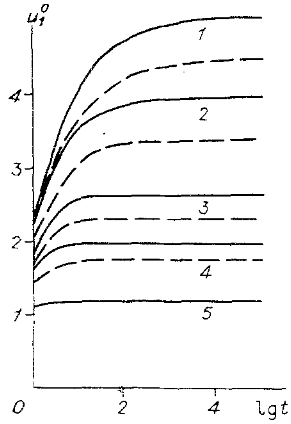

Fig. 2

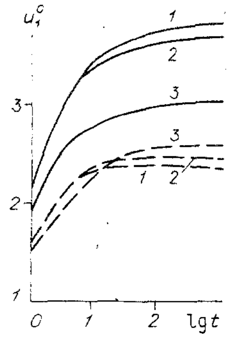

Fig. 3

In the general case we must solve the system (4), (5), and (8). The nature of the behavior of the dispersing plasma, as determined by comparison with the results of numerical calculations, can be assessed, however, by using the self-similar formulation or exact particular solutions of the respective problems.

We assume that motion with radiation loss occurs in the so-called regular regime and we look for the solution of a system in separated variables:

$$x_i = x_i^0(t)X_i(\xi_n), \quad u_i = u_i^0(t)U_i(\xi_n),$$
$$p = p^0(t)P(\xi_n), \; v = v^0(t)V(\xi_n), \; e = e^0(t)E(\xi_n). \tag{9}$$

It is natural to set $e^0 = p^0v^0/(\gamma - 1)$, $E = PV$. We assume that p^0, v^0, and e^0 are the thermodynamic parameters at the center of the volume under consideration, whereupon $P(0) = V(0) = E(0) = 1$. Without loss of generality, we assume that $x_i^0(0) = 1$ and then $X_i(\xi_n) = \xi_i$. Denoting $x_i^0(t) = \varphi_i(t)$, we obtain

$$x_i = \varphi_i\xi_i, \quad u_i = \xi_i\frac{d\varphi_i}{dt} = \frac{x_i}{\varphi_i}\frac{d\varphi_i}{dt},$$
$$\frac{\partial x_1}{\partial\xi_1}\frac{\partial x_2}{\partial\xi_2}\frac{\partial x_3}{\partial\xi_3} = \varphi_1\varphi_2\varphi_3, \quad \frac{\partial p}{\partial x_i} = \frac{\partial P}{\partial\xi_i}\frac{p^0(t)}{\varphi_i(t)}. \tag{10}$$

Thus, substituting (9) and (10) into (8) and (5), we have two systems of ordinary differential equations

$$\frac{de^0}{dt} + p^0\frac{dv^0}{dt} = -QC_e(e^0)^{-\alpha}(v^0)^{-\beta},$$
$$\varphi_i\frac{d^2\varphi_i}{dt^2}\frac{1}{p^0v^0} = C_i, \quad v^0 = \varphi_1\varphi_2\varphi_3, \quad e^0 = p^0v^0/(\gamma - 1) \tag{11}$$

to find the time dependence as well as to determine the parameters with respect to the Lagrange coordinate:

$$(PV)^{-(\alpha+1)}V^{-\beta} = C_e, \quad -V/\xi_i\partial P/\partial\xi_i = C_i. \tag{12}$$

In (11) and (12) C_e and C_i are separation constants. When we choose $C_e = 1$ the first equation of (12) reduces to the form $PV^n = 1$ $(n = 1 + \beta/(1 + \alpha))$. Since dispersal occurs in a vacuum and at the boundary of the considered $p = 0$, the second group of equations is solved with the boundary conditions

$$P(\xi_i)\Big|_{\sum_{i=1}^{3}\xi_i^2/\xi_{i0}^2=1} = 0. \tag{13}$$

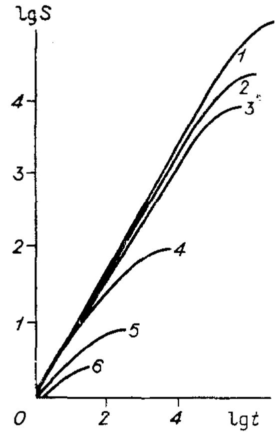

Fig. 4

From this it follows that $C_i = \frac{2n}{n-1}\frac{1}{\xi_{i0}^2}$. The distribution of the pressure, specific volume, and internal energy is found in analytical form (it is similar to that given in [3] for the problem of dispersal with heating)

$$P = (1 - x^2)^{n/(n-1)},\ V = (1 - x^2)^{1/(n-1)},$$
$$E = PV = 1 - x^2,\quad x^2 = \sum_{i=1}^{3} \xi_i^2/\xi_{i0}^2.$$

Here ξ_{i0} are the maximum dimensions of the ellipsoid along its axes. The velocities, as in [1-3], are distributed linearly with respect to the coordinates $u_i = u_i^0\xi_i/\xi_i$ (u_i^0 are the velocities of the edge of the ellipsoid along its axes). We rewrite the system, using dimensionless quantities that depend only on t, omitting the superscripts of p^0 and v^0 for convenience:

$$\varphi_i d^2\varphi_i/dt^2 = pvC_i,$$
$$\frac{dp}{dt} = -Q\frac{(pv)^{-\alpha}v^{-\beta}}{v}(\gamma - 1) - \frac{p}{v}\frac{dv}{dt}\gamma,\quad v = \varphi_1\varphi_2\varphi_3. \tag{14}$$

The results obtained by calculating system (14) for $\alpha = -2/3$ and $\beta = 2/3$ in the case of axisymmetric flow in which $\varphi_2 \equiv \varphi_3$, i.e., for a disk-shaped ellipsoid. The aim of the numerical calculations was to determine the dependence of the plasma-jet characteristics on Q and $\eta = \xi_3^0/\xi_1^0$ is the degree of ellipsoid compression. We note that luminescence does indeed intensify the directivity. The higher Q is, the more rapidly the pressure and internal energy decrease because of radiation and the gas manages to distill only along the minor axis. The explanation for this is that distillation along the major axis occurs only in the later stages, when the gas has already cooled.

The calculations were carried out for various Q and η. The coefficient $k = u_1^0/u_2^0$, i.e., the ratio of velocities along the minor and major axes, was introduced to characterize the directivity of the dispersal. The graph of k(t) is shown in Fig. 1 for $\eta = 10^4$ (solid lines) and 10^2 (dashed line). Curves 1-5 correspond to Q = 0.02, 0.2, 0.6, 1.0, and 2.0. We see that k decreases uniformly as Q increases. The gas cools very rapidly at $Q \gtrsim 2$ and as a result $k = \eta$. During the entire time, therefore, the dispersal takes place practically in the form of a flat disk. Calculations for other values of η showed that k grows uniformly with η at fixed values of Q. Accordingly, the asymptotic value of $k(\infty)$ is higher when η is larger.

Figure 2 shows the dependence of u_1^0 (dimensionless velocity at the edge of the ellipsoid along its minor axis) on t for $\eta = 10^4$ (solid lines) and 10^2 (dashed lines) for the same values of Q as in Fig. 1. The velocity u_1^0 decreases with growing Q. This is because of the decrease in thermal energy as a result of deexcitation and because of the decrease in the pressure and acceleration. Comparing the analogous functions $u_1^0(t)$ for various values of η at fixed values of Q, we find that at Q = 0.2 (Fig. 3, solid lines) u_1^0 increases uniformly with η. At Q = 0.6 (dashed lines) some nonuniformity arises. Lines 1-3 correspond to $\eta = 10^4$, 10, and 1. When the asymptotic curve is reached the velocity of a sphere (η = 1) is higher than that of a thin foil ($\eta \gg 1$) – the dashed lines in Fig. 3. In the initial stage of dispersal the velocity is higher for a thin foil than for a sphere and thus the sphere overtakes the foil in velocity. This is because the faster drop in density for the sphere decreases the intensity of luminescence at the same value of Q.

In this problem of the dispersal of an ellipsoid (different) pressure gradients exist along all of its axes; the asymptotic law (with respect to time) of the density variation for any ratios of the initial dimensions along different axes is the same and coincides with the law for a sphere ($\rho \sim 1/t^3$). Although the directivity effect does exist and is very pronounced for disk shape and elongated ellipsoids, the law whereby ρ decreases with t is the same and the gain can be by only a certain number of times. This is due to the nonuniform distribution of the initial density and pressure, including along the major axis.

At the same time, when a thin filament or flat foil is heated by a laser or electron beam with uniform irradiation the pressure along the major axis is the same along its entire central region for a fairly long time, until rarefaction waves arrive at the pertinent points from the edges of the disk. The flow in this stage is uniform, therefore in the entire central region of the filament or disk. At the same time, if the filament or disk is thin, the rarefaction wave travels rather quickly along the radius to the axis of the filament or along the entire thickness of the disk to its symmetry plane and a pressure and density gradient arises along the radius or in the direction perpendicular to the plane of the disk. This makes it possible to find the acoustic velocity distribution along the radius of the filament or the thickness of the disk as well as the law of its variation with time in a one-dimensional zone, i.e., the shape and temporal position of the boundary of the rarefaction wave. We can thus follow the process of disintegration of a one-dimensional zone. This was done for the adiabatic case by Smirnov [12], who noted that as the gas cools and the acoustic velocity decreases the advancement of the rarefaction wave slows down. Moreover, for a long filament the one-dimensional segment of dispersal can persist for the entire time, with its length increasing with the adiabatic index γ. Clearly, luminescence causes the gas to cool more rapidly, slows down the propagation of the rarefaction wave from the ends of the filament or from the edges of the disk, and increases the time for which a one-dimensional dispersal pattern exists.

The highest temperature and the highest acoustic velocity, according to (13), are reached in the plane of the disk. The size R_1 of the region occupied by one-dimensional flow is defined as

$$R_1 = \eta - S(t), \quad S = \int_0^t c^0(t)\,dt \tag{15}$$

(c^0 is the dimensionless acoustic velocity in the plane of the disk and S is the path traversed by a rarefaction wave in a time t). Figure 4 shows S(t) for Q = 0.01, 0.02, 0.03, 0.1, 0.2, 1.0 (lines 1-6). At large values of Q, i.e., under intensive luminescence, S becomes substantially shorter and the one-dimensional segment becomes longer. The luminescence thus also intensifies the directivity of the dispersal. For a wide plane disk ($\eta \gg 1$) in the case of strong luminescence the dispersal of the central part proceeds two-dimensionally, although more slowly than in the adiabatic case.

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LOCALIZED EXPLOSION IN A MATERIAL WITH A MAGNETIC FIELD AND THE CONSEQUENCES OF FINITE CONDUCTIVITY IN A MAGNETOHYDRODYNAMIC MODEL

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Introduction. An explosion in an empty space or a rarified gas in the presence of a magnetic field is the prototype of a number of natural cosmic and laboratory processes [1]; experimental explosions in the upper atmosphere [2, 3] have produced a stream of numerical and theoretical works. Magnetic retardation and conversion of plasma cloud energy with dispersion in an empty space has been considered in [4, 5]; in [5] this was done by numerical solution of two-dimensional gas dynamic equations. On the basis of a hybrid model a study was made of collisionless interaction with a magnetized material of unidimensional cylindrical [6] and two-dimensional "spherical" [7] plasma clouds. A unidimensional cylindrical explosion was computed in a magnetohydrodynamic approximation in [8].

Even without a magnetic effect a large scale explosion at a height is two dimensional due to the nonuniformity of the atmosphere over the vertical; detailed calculations are given in [9]. With the action of a magnetic field inclined to the vertical, flow becomes three-dimensional. Naturally, there is an increase in the difficulty of the calculation, and at the highest level the difficulty is aggravated for selecting a physical model (collision—collisionless flow, variability of ionization, etc.). Therefore, in order to understand these phenomena solutions for simple model problems which take account of some part of the actual features of the process are useful. For this purpose in the present work the following step is made compared with [8]: a "spherical" explosion is considered in an MHD-approximation. By means of appropriate averaging with respect to angles the two-dimensional problem in the case of a uniform material is converted to a unidimensional problem. Within the scope of sector [9] approximation the case is studied of a nonuniform atmosphere. In order to solve these problems a second order of accuracy scheme is used for the method of large particles with introduction of artificial viscosity. In conclusion the question is touched upon of refining the approximation of ideal conductivity and the conclusions which emerge as a result of this.

Approximate Reduction of the Two-Dimensional MHD-Problem to a Spherically Symmetrical Problem. We turn to gas dynamic description of motion without discussing the question here of its justification under specific conditions. When concerning this description there are no unconditional contradictions, even natural ionization of a material is sufficient so that the conductivity is assumed to be infinite. Then a magnetic field **H** in a gas moving with

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